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Growth theory in a Keynesian mode: some Keynesian foundations for new endogenous growth theory

After a lapse of almost twenty years, the theory of economic growth has once again become a "hot" topic among economists (see, for instance, the recent symposium on new growth theory in the *Journal of Economic Perspectives*, Winter 1994). This revival has come under the banner of "endogenous growth," the principal contribution of which has been to endogenize steady-state growth. This feature represents a desirable advance, one that readily appeals to Keynesian economists. However, despite this innovation, new growth theory remains entirely within the old growth neoclassical paradigm owing to the absence of aggregate demand considerations.

This paper shows how endogenous growth theory can be modified to incorporate Keynesian aggregate demand theoretic foundations. With regard to specifics, the paper identifies two critical Keynesian influences that are absent in neoclassical constructions of the growth process:

1. Capital accumulation is driven by investment, so that it is investment spending by firms that determines the rate of capital accumulation. This contrasts with the neoclassical perspective in which capital accumulation is driven by the savings behavior of households.
2. In equilibrium, the rate of output growth must equal the rate of aggregate demand growth, which implies that the rate of aggregate demand growth can potentially constrain the rate of output growth. Once again, this contrasts with the neoclassical perspective, which assumes a dynamic version of Say's law whereby demand automatically grows with output.

When the above features are combined with a Kaldorian (Kaldor,

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1957) technical progress function, the result is a Keynesian theory of growth in which the rate of aggregate demand growth affects the steady-state rate of output growth. The logic is as follows: aggregate demand growth affects investment spending, and investment spending affects the rate of technical progress; consequently, aggregate demand growth affects technical progress and output growth.

The structure of the paper is as follows. The next section briefly revisits old growth theory and examines its principal implications and limitations. The following two sections then examine the implications of modifying the old growth framework so as to make the rate of capital accumulation dependent on investment behavior and financial markets. It turns out that, although these are important changes, they are not sufficient to produce a Keynesian theory of growth. This is followed by an examination of the foundations of new endogenous growth theory that shows how a Keynesian theory of growth can be developed by placing the mechanisms of endogenous growth in an economy in which capital accumulation is driven by investment spending. The conclusion is that Keynesian growth theory requires *both* the mechanisms of endogenous growth and that capital accumulation be governed by investment spending (rather than saving).

Old growth theory revisited

The Solow (1956) growth model represents the paradigmatic model of old growth theory. The most important feature of the model is that the steady-state growth rate depends exclusively on the rates of population growth and labor augmenting technical progress, and as long as these variables are exogenous, steady-state growth is also exogenous.

A second feature of the model is that the rate of capital accumulation depends exclusively on household saving behavior and is independent of firms' investment spending. The model therefore assumes that the realization of savings is unproblematic, with increases in household saving being automatically translated into one-for-one increases in investment spending. This treatment disregards the fundamental concern of Keynesian economics with the investment–saving nexus, and it contrasts with Keynesian beliefs that it is investment behavior that determines the extent to which household savings are realized in the form of capital accumulation.

A third, and related, feature is that there is no mention of any demand constraints. Thus, the model implicitly embodies a dynamic version of

Say's law whereby all output growth is willingly demanded; growth of demand is deemed unproblematic, and demand expands *pari passu* with supply. This feature is deeply at odds with the Keynesian emphasis on aggregate demand. Within the short run, it is the "level" of demand that potentially constrains output and employment: Extrapolated to a growth context, it is the "rate of growth of demand" that may constrain the rate of output growth.

Although the Solow growth model embodies the core features of the neoclassical paradigm, it lacks any concern with monetary factors, and the real interest rate is determined exclusively by real factors. Tobin (1965) expanded the scope of the model by incorporating money and portfolio considerations. The motivation for this step was to show how the inclusion of money and portfolio choices affected the steady-state capital–labor ratio. In making this change, Tobin incorporated the Keynesian liquidity preference theory of interest rates within neoclassical growth theory, thereby making Keynesian monetary theory relevant for long-run economics.[1]

Though incorporating monetary factors, however, the Tobin growth model failed to endogenize steady-state growth, which remained determined by the exogenously given rates of population growth and technical progress. Moreover, capital accumulation continued to be driven by household savings behavior rather than firms' investment spending, and there was also no role for aggregate demand factors. In these regards, Tobin's monetary growth model remained similar to Solow's original neoclassical growth model.

Stage 1: Neoclassical growth models with an investment function

The above observations are revealing of the limitations of both neoclassical nonmonetary and monetary growth theory. In these models, the locus of capital accumulation is the household rather than the firm, and investment spending passively adjusts to ensure that all household saving is fully realized as new capital formation. This contrasts with Keynesian economics, which emphasizes the primacy of investment in determining capital accumulation.

[1] Tobin's (1965) approach to money and growth was macroeconomic in character in that it assumed the existence of a well-defined money demand function. Sidrauski (1967a, 1967b) adopted a microeconomic approach that sought to provide a microeconomic foundation for money based on the presence of money in either households' utility functions or firms' production functions.

To address this problem, the Solow growth model can be reformulated such that investment determines the rate of capital accumulation. This can be accomplished by respecifying the fundamental equation of motion governing the evolution of the capital–labor ratio and adding an equation determining investment spending per worker:

(1) $\dot{k} = I - [d + n + a]$ [Capital deepening]

(2) $I = z(r)f(k) \quad z_r < 0, f_k > 0, f_{kk} < 0$ [Investment function]

(3) $r = f_k$ [Interest rate]

(4) $g_y = n + a + [f_k k/f(k)]\dot{k}/k$ [Output growth]

where k = capital–labor ratio; I = gross investment per worker; d = rate of depreciation; n = rate of population growth; a = rate of labor augmenting technical change; z = marginal propensity to invest per worker; and r = real interest rate.

A dot above a variable denotes the time rate of change. Equation (1) determines the evolution of the capital–labor ratio. Equation (2) determines the flow of investment spending, with $f(k)$ being the intensive form production function. Equation (3) determines the interest rate, while equation (4) determines the rate of growth of output and is obtained from differentiation of the intensive form production function. The term $f_k k/f(k)$ represents the elasticity of output with respect to capital, which is constant under the assumption of a Cobb-Douglas production function.

The model is formally similar to the conventional Solow model, subject to the change that firms' investment spending determines the path of capital accumulation, with saving passively accommodating investment. Stability of the model requires that the rate of increase in investment spending decrease as the capital stock per worker increases. The stability condition is $d^2I/dk^2 = zf_{kk} + 2z_r f_{kk} f_k + f(k)z_{rr} f_{kk}^2 + f(k)z_r f_{kkk} < 0$. In this event, the steady-state capital–labor ratio and growth rates are given by

(5) $k^* = k(\overline{d}, \overline{n}, \overline{a})$

(6) $g_y = n + a.$

The Spartan character of the neoclassical growth model, with its lack of a financial sector, makes it difficult to incorporate investment behavior in a sensible fashion. Per equation (2), investment spending is negatively related to the interest rate, which is sensible. However, the interest rate equals the marginal product of capital, which means that investment spending is negatively related to the marginal efficiency of capital; this is not sensible. Incorporating investment as the determining factor behind capital accumulation therefore implicitly calls for a theory of interest rates in which the interest rate can be detached from the marginal efficiency of capital. In this case, increases in the marginal efficiency of capital can act as a spur to investment, while the interest rate acts as a restraint.

A second failing of the model is the absence of any references to aggregate demand (AD) in the growth process. The natural channel for incorporating a role for AD growth is to have it influence investment:

(7) $$I = z(r, g_D)f(k) \qquad z_r < 0, \qquad z_{gD} > 0,$$

where g_D is the rate of AD growth. AD growth therefore exerts a positive influence on the flow of investment spending per worker. The introduction of AD growth then calls for a theory of AD growth, and it must also explain how AD growth is brought into balance with output growth. Keynesian economics has long had a theory of the "level" of AD and a theory of adjustment of the level of output, but it lacks a theory of AD growth.

Finally, as long as the steady-state rate of output growth is exogenous, and steady-state AD growth must equal steady-state output growth, then AD growth is ultimately ruled by the rate of steady-state output growth. Thus, if AD growth is to have an independent influence on the steady-state rate of output growth, the latter must be endogenized. A theory of endogenous output growth is therefore a prerequisite for developing a meaningful Keynesian theory of growth; otherwise, the rate of supply growth rules the roost.

To sum up, growth theory in a Keynesian mode calls for four essential modifications to the neoclassical model: First, the introduction of an investment function making firms the locus of capital accumulation; second, the introduction of a theory of interest rates in which the marginal efficiency of capital is a spur to investment, and the rate of interest a restraint; third, the construction of a theory of aggregate demand growth; fourth, endogenizing the rate of output growth in a fashion that allows aggregate demand growth to affect it directly or indirectly.

Stage 2: Neoclassical growth with an investment function and financial markets

As noted above, the Spartan character of the neoclassical growth model makes it hard realistically to incorporate investment. Tobin's (1965) monetary growth model is better positioned in this regard. Within that model, agents can accumulate wealth in the form of either money or capital, and this represents the rudimentary beginnings of a financial system. However, households are still the locus of capital accumulation since decisions to save in the form of capital are fully realized.

Introducing an investment function introduces a distinction between households and firms, and allows firms to control the rate of capital accumulation, while the existence of asset markets means that these markets can serve to determine the cost of capital for firms. The household–firm distinction is reflected in the owner–manager distinction; it is also reflected in asset markets by the distinction between equities and capital. The owner–manager distinction calls for separate representations of the behaviors of households and firms, with firms determining the pace of investment spending and capital accumulation, while households determine the valuation of financial claims in asset markets. Capital is held by firms, while equities are held by households. The latter represent the ownership of firms' capital, and their valuation depends on households' expectations of firms' performance and profitability. This valuation is important if it affects household saving behavior, or if it affects firms' investment spending. Incorporating an investment function and asset markets into the neoclassical growth model therefore raises multiple issues about the behavior of firms, the workings of asset markets, and the effect of asset market prices on investment spending.

An illustrative possibility that is broadly neo-Keynesian in character is as follows:

(8) $\dot{k} = I - [d + n + a]k$ [Capital deepening]

(9) $I = I(f_k/R) \quad I' > 0$ [Investment function]

(10) $R = x_1 r_E \quad 0 < x_1 < 1$ [Managerial cost of capital]

(11) $P_E E = E^*(r_E, p, k, x_2) \quad E^*_{rE} > 0, E^*_p > 0, E^*_k > 0, E^*_{x2} < 0$ [Equity market clearing]

(12) $M = M^*(r_E, p, k, x_2)$ $M^*_{rE} < 0, M^*_p < 0, M^*_k > 0, M^*_{x2} > 0$ [Money market clearing]

(13) $r_E = f_k k / P_E E$ [Market cost of capital]

(14) $g_M = m - p - n$ [Rate of real money growth]

(15) $g_y = n + a + s_k \dot{k}/k$ [Rate of output growth]

(16) $p = m - g_y$ [Rate of inflation]

where R = managers' cost of capital; r_E = rate of return on equities; x_1 = managerial discount factor applied to r_E; P_E = market price of equities; E = Units of equity in issue per worker; p = rate of inflation; M = real money balances per worker; E^* = equity demand per worker; M^* = real money demand per worker; x_2 = illiquidity discount applied by shareholders against equities; g_M = growth of real money balances per worker; g_y = rate of output growth; m = rate of nominal money supply growth; and s_k = capital's share of output.

Equation (8) determines the evolution of the capital–labor ratio, while equation (9) determines gross investment spending per worker. The latter is a positive function of the ratio between the marginal efficiency of capital and managers' cost of capital; this specification bears some resemblance to Brainard and Tobin's (1968, 1977) "q" theory of investment. Equation (10) determines the cost of capital adopted by managers, which is equal to the market rate of return on equities adjusted by a managerial discount factor. This adjustment factor is discussed further below.

Equations (11) and (12) represent the equity and money market clearing conditions, respectively. The introduction of an investment function and the accompanying distinction between households and firms forces us to distinguish between physical capital and equities: this distinction is absent in neoclassical monetary growth theory, which lacks an investment function and has capital accumulation directly driven by household portfolio demands. Equity markets determine equity prices, and these adjust to ensure that wealth owners hold their desired amounts of wealth in equity form. The demand for equities is subject to an illiquidity discount (Kaldor, 1960) that reflects the fact that equities are less liquid than money, and the magnitude of the discount varies with shareholder liquidity preference. The significance of the illiquidity discount is discussed further below. Equation (13) determines the rate of return on equities given firms' profits and equity prices. Equations (14) and (15) determine the rates of real money and output growth. Finally, equation (16) determines the rate of inflation, which is equal to the rate of nominal money growth minus output growth: for simplicity, velocity is assumed constant.[2]

With regard to the specification of household portfolio demands, these depend on the return on equities, the rate of inflation, and the capital stock per worker, which proxies for income per worker. Note that, in equation (11), equity prices affect both the nominal supply of equities and the demand for equities through their effect on the rate of return on equities.

The distinction between owners (households) and managers (firms) and between equities and physical capital introduce two important loose linkages. The owner–manager distinction means that managers control firms and the investment decision, and the level of investment spending depends on managers' cost of capital. This cost is the market rate of return on equities, adjusted by a factor x_1. If $x_1 < 1$, this implies that managers have a lower discount rate than owners. The managerial discount factor therefore proxies for the extent to which managers depart from the strict profit maximization predicated on an identification with shareholders' required rate of return, and overaccumulate capital by adopting a cost of capital below that determined by financial markets. This issue has recently been addressed by Crotty (1990).

The equity–physical capital distinction represents a second loose linkage, with owners valuing equities in equity markets. This valuation depends on the underlying profit stream, $f_k k$, but it is also affected by the illiquidity discount factor, x_2, which affects the demand for equities. In the current model, equity holders see through to the underlying value of the firm as determined by $f_k k$, but another way in which the equity–physical capital distinction might play out is if equity holders have prospective valuations that systematically differ from this.

The wealth constraint at each moment in time implies that

$$P_E E + M = E^*(\cdot) + M^*(\cdot), \tag{17}$$

which upon rearranging becomes

[2] MV can be identified with nominal demand, so that the growth of MV represents nominal demand growth. In this neo-Keynesian model, nominal demand growth is therefore exogenous, and simply determines the steady-state rate of inflation contingent on the steady-state rate of output growth.

$$[P_E E - E^*()] + [M - M^*()] = 0. \tag{17'}$$

Consequently, when either the equity market or money market clears, the other market also clears. The dynamic equations determine k, g_M, g_y, and p, so that the state variables are the capital–labor ratio, real balances per worker, output, and the general price level. Given these state variables, the model then determines r_E, P_E, R, and I at each moment in time.

The stability of the model depends exclusively on the evolution of the capital–labor ratio, which also drives output growth, inflation, and the growth of real balances. As k increases, this drives down the marginal efficiency of capital, which slows investment spending. However, increases in k have an ambiguous effect on equity yields, which determine the cost of capital. Increased k raises income and the demand for equities, which drives up equity prices and drives down equity yields; balancing this, increases in k raise profit streams, which drives up the yield on equities. Stability therefore requires that $dI(f_k/R)/dk > 0$ and $d^2I(f_k/R)/dk^2 < 0$; given this, investment per worker increases as k increases, but it increases at a diminishing rate, thereby preventing an explosion in k. The diagrammatic representation of stability is presented in figure 1.

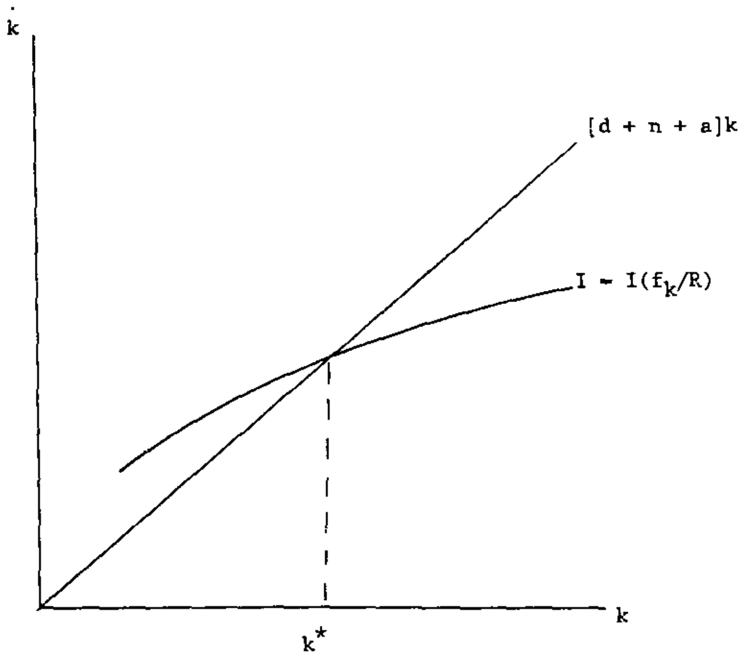
In steady state, the system simplifies to a two-equation system given by

$$k = I(\overset{+}{f_k/x_1 r_E})/[d + n + a] \tag{18}$$

$$f_k k/r_E = E^*(\overset{+}{r_E}, \overset{+}{m - n - a}, \overset{+}{k}, \overset{-}{x_2}) \tag{19}$$

The endogenous variables are k and r_E. The introduction of the owner–manager and equity–physical capital distinctions introduces a distinction between the marginal product of capital and the rate of return on equities. Managerial behavior influences capital accumulation through the factor x_1, while shareholders' liquidity preference affects the cost of capital through the factor x_2. Totally differentiating equations (18) and (19) and arranging in matrix form yields

Figure 1 Determination of the steady-state capital–labor ratio in a neoclassical growth model with investment and financial markets



$$\begin{vmatrix} 1 - I'f_{kk}/x_1 r_E [d+n+a] & I'f_k/x_1 r_E^2 [d+n+a] \\ [f_{kk}k + f_k]/r_E - E_k & -f_k k/r_E^2 - E_{rE} \end{vmatrix} \begin{matrix} dk \\ dr_E \end{matrix}$$

$$= \begin{vmatrix} -I'f_k/r_E x_1^2 [d+n+a] & 0 & 0 \\ 0 & E^*_{x2} & E^*_{m} \end{vmatrix} \begin{matrix} dx_1 \\ dx_2 \\ dm \end{matrix}$$

If $[f_{kk}k+f_k]/r_E - E_k > 0$, then the Jacobian is unambiguously negative. This assumption implies that an increase in the capital stock raises discounted corporate profits (i.e., the effective supply of equity wealth) more than it increases equity demand so that the market yield on equities rises.

Given this, the comparative statics are:

$$dk/dx_1 < 0,\ dk/dx_2 < 0,\ dk/dm > 0.$$

Increases in the managerial adjustment factor raise the required return on capital and lower the capital–labor ratio. Increases in liquidity preference cause a shift out of equities into money, which lowers equity prices and raises the required market rate of return, thereby getting managers to reduce capital accumulation. Finally, increased nominal money growth increases inflation, which prompts households to shift toward equities, lowers the required market rate of return, and raises capital accumulation by managers. The effects on the market rate of return are:

$$dr_E/dx_1 < 0, \; dr_E/dx_2 > 0, \; dr_E/dm < 0.$$

Although the introduction of an investment function and financial markets, distinguishing between equities in household portfolios and physical capital held by firms, captures the institutional realities of capitalist economies, the steady-state rate of growth remains exogenously determined by the rates of population growth and technical progress, and is independent of aggregate demand concerns. Augmenting the neoclassical model to include an investment function and financial markets is therefore insufficient to produce a Keynesian theory of growth. Still missing is a mechanism that allows aggregate demand to affect the steady-state rate of growth.

Stage 3: The mechanisms of endogenous growth

A key feature of neoclassical growth theory is that the steady-state rate of growth is exogenously determined. This exogeneity stymied the old growth research program, and greatly diminished its policy content. The principal contribution of new endogenous growth theory has been to resolve this impasse by introducing a range of mechanisms that render steady-state growth subject to endogenous variation. The key innovation involves respecifying the process generating technical change so as to allow it to depend on the decisions of economic agents.

Within the United States, endogenous growth theory has emphasized knowledge and human capital formation. Romer (1986) introduces knowledge externalities into the aggregate production function that promote accelerating knowledge acquisition and growth. Lucas (1988) emphasizes the role of human capital in the growth process, and this necessitates introducing human capital as an additional argument in the production function. Endogenous growth emerges when the aggregate stock of human capital is allowed to have an external effect on the rate of technical change as in Romer (1990).

The British variant of endogenous growth has emphasized investment in physical capital. Drawing on a line of reasoning pioneered by Kaldor (1957) and Kaldor and Mirlees (1962), Scott (1989) suggests that endogenous growth operates through the effects of investment spending on the flow rate of technological innovation, with technical progress being the endogenous product of capital accumulation.[3] Technical progress is therefore both "revealed" and "realized" through investment, so that investment serves simultaneously as the means of (1) expanding the capital stock, (2) feeding technical innovations into the production process, and (3) uncovering further possibilities for innovation. Expanding the capital stock is the traditional interpretation of investment; feeding innovations into the capital stock is the "vintage" approach to investment; opening possibilities for further technical advances is the endogenous growth interpretation of investment.

The mechanism of endogenous growth can be readily understood from the following specification of the technical progress function:

$$a = Ak^bI^c \qquad A > 0. \tag{20}$$

Equation (20) determines the rate of labor augmenting technical progress as a positive function of the capital–labor ratio and the rate of capital accumulation per worker. Nested inside (20) is the standard case of exogenous technical progress, which occurs if $b = c = 0$. If $b = 0$, then it is only the "flow" of investment spending per worker that affects the rate of technical advance: if $c = 0$, it is only the current "stock" of capital per worker that has an effect.[4] The critical feature about equation (20) is that the rate of technological progress is now endogenously determined; this is the core innovation behind endogenous growth theory.

Unfortunately, Kaldor (1957) used a linear technical progress function that was equivalent with an underlying Cobb–Douglas production function in which the steady-state rate of output growth was independent of the investment–output ratio. The above specification avoids this problem. Neoclassical endogenous growth models use similar stock-flow specifications, albeit emphasizing human capital facets rather than physical investment. In Romer (1986), it is the stock of knowledge that

[3] Another dimension to Scott's (1989) work is the issue of growth accounting and measurement of capital. This latter issue is not addressed in the current paper.

[4] Stock effects may be important because they introduce increasing returns to the growth process, and this can explain the nonconvergence of cross-country growth rates. A possible microeconomic rationale of their effect is that more capital per worker yields more opportunities for seeing where innovations are possible.

matters, while, in Romer (1990), the flow expenditure on R&D interacts positively with the existing stock of knowledge. However, though potentially compatible with a range of macroeconomic paradigms, thus far the technical progress function has been exclusively placed within the context of neoclassical growth models in which savings drives capital accumulation, and the effects of aggregate demand growth are absent.[5] Thus, existing new endogenous growth literature remains severely non-Keynesian.

Equation (20) is a reduced-form specification. The microeconomics of why investment spending affects the rate of technical advance are detailed in Scott (1989). From a policy standpoint, the important implication is that the rate of technical progress can be influenced by policies that affect either the capital stock per worker or the flow of investment per worker. Exactly the same considerations apply for representative agent choice–theoretic endogenous growth models that rely on knowledge and R&D expenditures (e.g., Romer, 1990). In these models, R&D spending affects the growth rate, and policies or institutional arrangements that affect R&D spending therefore affect the equilibrium growth rate. Thus, such models implicitly embody a Kaldorian technical progress function in which the the symbols k and I are replaced by the stock and flow of R&D. This reveals how Kaldor (1957) is the progenitor of endogenous growth.

Stage 4: Introducing aggregate demand growth

Constructing a Keynesian theory of growth requires combining a technical progress function with an investment function in which investment spending is driven by aggregate demand. It is only when these features are joined that a Keynesian model of growth emerges: The former endogenizes the equilibrium growth rate, while the latter provides a conduit for aggregate demand to affect output growth. Harrodian growth theory (Harrod, 1939) allowed aggregate demand factors to cause fluctuations around the natural (supply-side) rate; Kaldorian endogenous growth theory allows aggregate demand factors to affect the natural rate directly, thereby enabling interaction between the growth of demand and the growth of supply. The balance of this section presents a Keynesian growth model that incorporates this feature.

[5] Palley (1994) provides a survey showing how the mechanisms of endogenous growth can be grafted onto earlier neoclassical growth models.

The model developed below can be viewed as a growth theoretic analogue of the Keynesian income–expenditure model. For simplicity, it excludes the financial market considerations addressed in the previous section. Adding these considerations would produce a growth theoretic analogue of the IS/LM model. Within such a model, financial markets would matter for growth through their impact on the market cost of capital, and through the distinction between managerial and owner behavior. Both of these features influence the extent of capital accumulation by firms, and this in turn affects the steady-state rate of output growth through the technical progress function.

The equations of the model are:

(21) $I = z(g^d) \quad z_{gd} > 0$ [Investment function]

(22) $k = I - [d + n + a]k$ [Capital deepening]

(23) $g_y = n + a + s_k k/k$ [Output growth]

(24) $a = a(k, I) = a(k, g^d) \quad a_k > 0, a_{gd} > 0$ [Technical progress function]

(25) $\dot{g}^d = G(g_y - g^d) \quad G' > 0$ [Demand growth adjustment]

Equation (21) is the investment function, which replaces the saving function as the determinant of capital acccumulation. A critical aspect of this specification is that investment spending is positively related to the growth of aggregate demand, with firms expanding capacity to meet growing demand. This represents a form of accelerator model. If financial markets were included, then the cost of equity capital and managers' adjustment factor would both enter as arguments determining the flow of investment spending.

Equation (22) determines the evolution of the capital–labor ratio, equation (23) determines the rate of output growth, while equation (24) is the mechanism of endogenous growth. According to this mechanism, both the capital stock per worker and the flow of investment per worker positively affect the rate of labor augmenting technical progress.

Equation (25) determines the evolution of the rate of aggregate demand growth, which responds positively to the rate of output growth. For an equilibrium to exist, demand growth must ultimately equal the rate of output growth, or else the economy would be characterized by ever expanding excess demands or supplies. In the current formulation, aggregate demand growth is assumed to respond positively to output growth. This represents what may be termed the case of "optimistic Keynesian dynamics" (Palley, forthcoming). An alternative case of "pessimistic Keynesian dynamics" is when aggregate demand growth responds negatively to output growth.

By a process of substitution, equations (21) through (25) can be reduced to a two-equation system given by

$$\dot{k} = z(g^d) - [d + n + a(k, g^d)]k; \qquad (26)$$

$$\dot{g}^d = G(n + a(k, g^d) + s_k z(g^d)/k - s_k[d + n + a(k, g^d)] - g^d). \qquad (27)$$

Linearizing around the local equilibrium values and arranging in matrix form yields:

(26′), (27′)

$$\begin{vmatrix} \dot{k} \\ \dot{g}^d \end{vmatrix} = \begin{vmatrix} z_{gd} - a_{gd}k & -a_k k - d - n - a \\ G'\,[a_{gd} + s_k z_{gd}/k - s_k a_{gd} - 1] & D \end{vmatrix} \begin{vmatrix} g^d - g^{d}* \\ k - k^* \end{vmatrix}$$

where $D = G'[a_k + s_k z(g^d)/k^2 - s_k a_k] > 0$. The stability conditions are

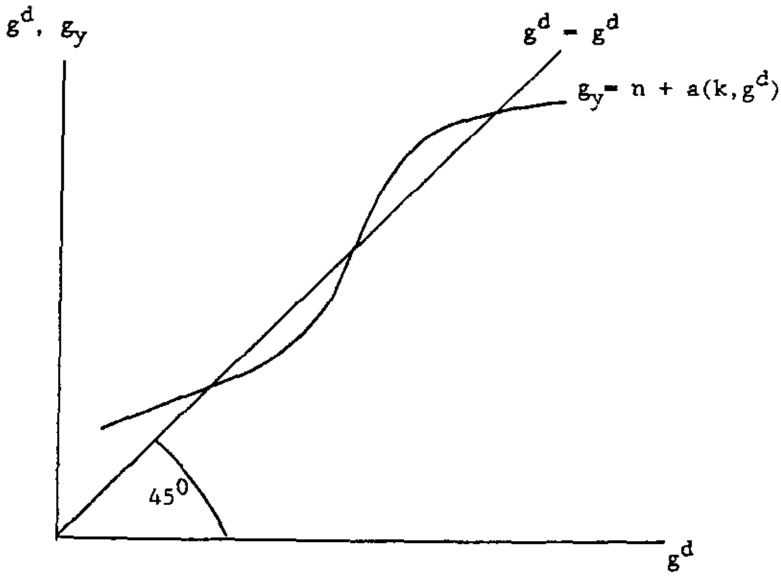
$$z_{gd}k - a_{gd}k + D < 0; \qquad (28)$$

$$[z_{gd} - a_{gd}k]D + G'[a_{gd} + s_k z_{gd} - s_k a_{gd} - 1][a_k k + d + n + a] > 0, \qquad (29)$$

which may or may not be satisfied. The logic of potential instability is readily understandable and rests on the interaction between the process of capital deepening and demand growth. Thus, a positive shock to the rate of demand growth could accelerate the process of capital deepening, thereby accelerating the pace of technical advance and output growth. Per equation (25), this would then accelerate demand growth, giving rise to the potential for a cumulatively unstable process. Such instability can be viewed as the dynamic analogue of multiplier instability in the static income–expenditure model. In the latter, stability requires that induced increases in the level of demand be less than the initial increase in income: In the current Keynesian growth model, stability requires that the induced increase in the growth of demand be less than the initial increase in output growth.

Figure 2 illustrates the possibility of multiple equilibria, with the outer equilibria being stable and the inner equilibrium being unstable. From a policy standpoint, the existence of multiple local equilibria is interest-

Figure 2 Determination of the equilibrium rate of growth in a Keynesian growth model in which the growth of demand affects investment spending and technical progress



ing because it means that macroeconomic policies that affect the rate of growth of demand may be able to shift the economy from a low-growth equilibrium to a high-growth equilibrium. However, unlike the static Keynesian macro model in which aggregate demand management can continuously shift the equilibrium, the requirement that equilibrium growth of demand equal growth of supply restricts demand growth to be consistent with supply growth as determined by the technical progress function and population growth.

Stage 5: Introducing excess demands

The above model focused on the effects of aggregate demand growth on output growth. However, equality of the "growth" of demand and output does not ensure balance between the level of demand and level of output, and this opens the possibility of persistent excess demand. Neoclassical growth models examine the process of economic growth under the assumption that markets clear so that there is (a) full employment, (b) balance between the growth of demand and growth of supply, and (c) balance between the level of demand and level of supply. In practice, the world appears very different, and growth frequently takes place under conditions of persistent excess demand or excess capacity. This point has been made by Nell (1991, 1994), who observes that excess demand represented the normal condition of the old eastern European command economies, while excess capacity represents the normal condition of capitalist economies.

Recognizing this possibility gives rise to a further extension of Keynesian growth theory whereby excess demand can persist along the equilibrium growth path, and it can also influence the rate of growth. The key insight is that excess demand influences the rate of growth through its effect on "incentives" facing economic agents. This effect is felt across different dimensions of the growth process, and it is useful to distinguish between "intensive" and "extensive" growth.

Intensive growth refers to growth achieved by improvements in organizational and engineering technology, holding inputs of capital and labor constant; it therefore corresponds to growth by innovation and invention. Extensive growth refers to growth through increased capital and labor inputs, holding technology constant; it therefore corresponds to growth by replication.

This distinction is important because excess demand conditions may differentially affect the processes of intensive and extensive growth. Positive excess demand has a positive effect on the rate of extensive growth. Firms are short of capacity, and this provides an incentive to build more; profits can be achieved through the easy channel of expansion by replication. These arguments are reversed for conditions of excess supply, when firms have an incentive to reduce the rate of replication and even cut back capacity as a means of reducing overhead and saving on capital costs.

When it comes to intensive growth, excess demand conditions may have a negative or a positive impact. Tight product market conditions mean that it is a seller's market, and this reduces the need to innovate. The cost of innovation is also high since resources are expensive and innovation means risking a sure outcome for an unsure one. However, moderate shortages of capacity may provide a spur to using existing capacity more efficiently. Balancing this, extreme positive excess demand may promote managerial x-inefficiency, since managers are protected from market discipline because customers remain loyal and take whatever they can get. Similarly, positive excess demand may reduce the incentive for workers to engage in productivity enhancing innovations; with jobs plentiful, the ongoing beneficial effects of labor market discipline are reduced. By contrast, whereas excessively strong market conditions may discourage intensive growth, weak market conditions may give firms an incentive to reduce costs. In such an environment, the firm's very survival may be threatened, and lowering costs enables the firm to lower prices and gain market share at the expense of rival firms.

The above arguments regarding the growth effects of excess demand can be readily included in the Kaldorian technical progress function, which now becomes

(30) $$a = Ak^b I^c E^e \; A > 0, a_k > 0, a_I > 0, a_E \gtrless 0,$$

where E = excess demand measured as the demand–capacity ratio. The sole change from the earlier specification is the inclusion of an excess demand effect on technical progress.[6]

This representation of technical change can now be placed within the above Keynesian growth model. The equations of the new model are:

(31) $I = z(E, g^d)$ [Investment function]

(32) $\dot{k} = I - [n + a + d]k$ [Capital deepening]

(33) $g_y = n + a + s_k \dot{k}/k$ [Output growth]

(34) $a = a(k, I, E) \quad a_k > 0, a_I > 0, a_E \gtrless 0$ [Technical progress function]

(35) $\dot{g}^d = G(g_y/g^d) \; G' > 0, G(1) = 0$ [Demand growth adjustment]

(36) $E = D/Y$ [Excess demand]

[6] The measure of excess demand is formulated by reference to capacity rather than output. If it were formulated by reference to output, this would be problematic, as it would have implications for inventories. Thus, if output exceeded demand, firms would be producing more than they were selling. If output were storable, inventories would be persistently rising, raising the question of why firms do not cut output. If output were nonstorable, this would imply persistent wastage, again raising the question of why firms do not cut output. Specifying the degree of market imbalance by reference to capacity avoids this problem. Output can be less than capacity, and to the extent that capacity is elastic through such measures as varying hours, output can persistently exceed capacity.

(37) $\dot{E} = g^d - g_y$ [Excess demand adjustment]

There are three changes from the earlier model. First, investment spending is now a positive function of the level of excess demand, E, reflecting the impact of excess demand on extensive growth. Second, the rate of technical progress is also a function of E, though the sign is ambiguous. Third, the rate of change of demand growth is a function of the ratio of output growth to demand growth: this is a technical adjustment that is needed to undertake stability analysis.

Substituting (31) into (34) yields:

(38) $$a = a(k, z(E, g^d), E) = a(k, g^d, E),$$

and differentiating with respect to E yields:

$$da/dE = a_z z_E + a_E \gtrless 0.$$

The effect of excess demand on the rate of technical progress is therefore ambiguous. The first term is positive, reflecting the effect of excess demand on investment spending and extensive growth; the second term is ambiguous, reflecting the uncertain indirect effect on intensive growth. It transpires that this ambiguity is of key importance for macroeconomic growth policy since there may be regions in which strengthening demand conditions increases growth, and regions beyond which it lowers growth. Theoretical economists too quickly adopt the assumption of monotonicity; in practice, the world may be less cooperative.

By a process of substitution, this system of equations can be reduced to a dynamic three-equation system given by

(39) $$\dot{k} = z(E, g^d) - [n + a(k, g^d, E) + d]k;$$

(40) $$\dot{g}^d = G(\{n[1 - s_k] + a(k, g^d, E)[1 - s_k] + s_k z(E, g^d)/k\} / g^d);$$

(41) $$\dot{E} = g^d - n[1 - s_k] - a(k, g^d, E)[1 - s_k] - s_k z(E, g^d)/k.$$

This system can be linearized around a local equilibrium. As a three-dimensional system, the stability conditions are complicated expressions that may or may not be satisfied. Once again, the economic logic behind the potential for instability is readily understandable and rests on the interaction between the process of capital deepening and the evolution of demand growth. Thus, a shock to the rate of demand growth increases investment and increases the level of excess demand, thereby accelerating the pace of technical advance and output growth. This in turn accelerates demand growth, giving rise to the potential for a cumulatively unstable process. Note that if excess demand has a positive direct effect on technical progress, this increases the likelihood of instability.

From a policy perspective, the interesting feature of this model concerns the question of whether policy-sponsored variations in the level of excess demand, achieved through traditional aggregate demand management, can be used to influence the steady-state rate of growth. In equilibrium, the level of excess demand, the capital–labor ratio, and the rate of demand growth are all constant. Setting $\dot{E} = \dot{k} = 0$ and $g^d = g_y$, then, implies

$$z(E, g^d) - [n + a(k, g^d, E) + d]k = 0; \tag{42}$$

$$\text{n} + a(k, g^d, E) - g^d = 0. \tag{43}$$

Totally differentiating equations (42) and (43) with respect to k, g^d, E, and n, and arranging in matrix form, yields

$$\begin{vmatrix} z_{gd} - a_{gd}k & -a_k k - n - a - d \\ a_{gd} - 1 & a_k \end{vmatrix} \begin{vmatrix} dg^d \\ dk \end{vmatrix} = \begin{vmatrix} -z_E + a_E k & k \\ -a_E & -1 \end{vmatrix} \begin{vmatrix} dE \\ dn \end{vmatrix}$$

The Jacobian is given by $|J| = a_k[z_{gd} - a_{gd}k] + [a_{gd} - 1][a_k k + n + a + d]$. Assuming the Jacobian to be negative, the effect of an increase in the rate of population growth is

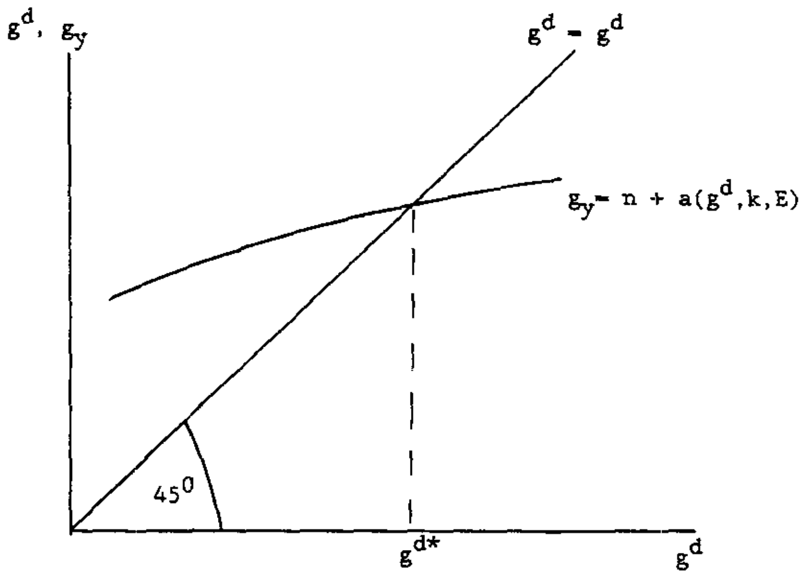
$$dg^d/dn = -[n + a + d]/|J| > 0.$$

The effect of a change in the level of excess demand on the steady-state rate of growth of output is given by

$$dg^d/dE = \{a_k[a_E k - z_E] - a_E[a_k k + n + a + d]\}/|J| \gtrless 0.$$

The reason for this ambiguity is the ambiguous effects of excess demand on technical progress. If excess demand has a positive effect, owing to a positive impact on both extensive and intensive growth, then the numerator will be positive, and the entire expression will be positive. The reverse holds if excess demand has a negative effect on technical progress.

Figure 3 Determination of the equilibrium rate of growth in a Keynesian growth model in which the growth of demand and the level of excess demand affect investment spending and technical progress



This situation is illustrated in figure 3, which shows the determination of equilibrium between the rate of growth of demand and output. The figure is analogous to a growth theoretic income–expenditure diagram. If excess demand has a positive effect on output growth, then an increase in excess demand will shift the output growth function upward, resulting in an equilibrium with higher steady-state output growth. However, if excess demand has a negative effect, the output growth function shifts down in response to an increase in the level of excess demand, resulting in lower steady-state equilibrium growth. Ultimately, resolving this problem is an empirical issue.

From the standpoint of welfare analysis for policy, the case when excess demand has a negative effect is the most problematic. This is because it poses a trade-off between "cake today versus more cake tomorrow." In this instance, stimulating the economy by raising the level of demand will result in more output and employment today, but since it raises the level of demand pressure and capacity utilization, it also lowers the rate of growth and future available output. Which path is preferred, therefore, depends on the social rate of discount. Note, if excess demand has a positive effect, then there is no trade-off, and society can have more cake today and more tomorrow. However, this claim is subject to the caveat that, at some high level of demand pressure, excess demand likely has a negative effect; this is the implication of the experience of the old Soviet-style economies.

The above model is for a closed economy. Open-economy considerations can be tentatively introduced by distinguishing between the growth of domestic and foreign (export) demand. This requires adding another equation defining aggregate demand growth and respecifying equation (35):

(44) $$g^d = H(g_D{}^d, g_F{}^d) \qquad H_{gD} > 0, H_{gF} > 0;$$

(35′) $$\dot{g}_D{}^d = G(g_y/g^d)$$

where $g_D{}^d$ = growth of domestic demand and $g_F{}^d$ = growth of foreign demand. Foreign demand growth is exogenous and positively influences aggregate demand growth. If an economy is stuck in a low-growth equilibrium with insufficient domestic demand growth, foreign demand growth can potentially serve to shift the economy to a high-growth equilibrium. In addition, shocks to the level of export demand raise the level of excess demand. In accordance with static Keynesian theory, this raises the level of output; the effect on growth depends on how excess demand affects steady-state growth, as discussed above.

Another issue raised by the model is inflation. The real effects of inflation operate through financial markets, and this necessitates an expansion of the model as outlined earlier. This aside, there are also unresolved theoretical issues regarding the determination of inflation. A general specification is:

(45) $$p = g^d - g_y + hE \qquad h \geq 0.$$

This allows both the growth of demand and the level of excess demand to affect steady-state inflation. However, it is theoretically plausible that the coefficient h is zero, so that the level of excess demand (measured by capacity utilization) has no impact on inflation. Instead, its impact may be felt exclusively on the distribution of income between wages and profits. This consideration begins to introduce additional Post

Keynesian concerns with the growth process, since income distribution may affect both the level of excess demand and the growth of demand.

Finally, the fact that excess demand conditions affect the equilibrium rate of growth is suggestive of why growth rates will be stochastic and hysteretic in character. The logic is as follows: Stochastic disturbances to the level of macroeconomic activity affect the level of excess demand, and this then affects the growth rate and renders it too stochastic. If the level of macroeconomic activity is also hysteretic in character, then disturbances to the level of excess demand will permanently shift the growth rate so that it too will exhibit hysteresis.

Conclusion

The developments associated with endogenous growth theory have reawakened interest in the theory of economic growth. However, new endogenous growth theory has been developed exclusively in the context of models that continue to rely on the neoclassical foundations of old growth theory. In particular, the process of capital accumulation continues to be based on household saving behavior, and economic growth remains unaffected by either the rate of demand growth or the level of excess demand. This paper has shown how endogenous growth theory can be used to develop a Keynesian theory of growth. Such a development involves two steps. The first involves incorporating the mechanisms of endogenous growth, thereby allowing for endogenous variation in the rate of growth. The second involves recognizing that capital accumulation is driven by firms' investment spending rather than household saving behavior. The inclusion of an investment function then creates a point of entry for aggregate demand factors. Together, these innovations allow aggregate demand factors to affect investment spending, which then affects the rate of growth.

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